Higher Loday algebras

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OUTLINE

Higher (Lie and) Loday (Leibniz) algebras:

1. Coalgebraic approach
2. Category theoretical standpoint
3. Operadic view
4. Supergeometric framework

LIE AND LODAY INFINITY CATEGORIES
COALGEBRAIC APPROACH

$$(L, d) \underset{\overset{i}{\leftarrow}}{\overset{p}{\rightarrow}} (V, d')$$

$$pi = \mathrm{id}_V,\; ip \overset{h}{\sim} \mathrm{id}_L$$

$$(L, d) : \mathrm{DGLA} \Rightarrow (V, d') : \mathrm{L}_\infty \text{ algebra}$$

THEOREM

(L, d) and (V, d') homotopy equivalent ds. An L_∞ structure on L induces an L_∞ structure on V.

$\rightsquigarrow$ Importance of L_∞ in BRST, closed string theory

$(V, [-,-])$: GLA, $\pi \in V^1$, $[\pi, \pi] = 0$

$\mathfrak{g} := \nu V[[\nu]]$, $(\mathfrak{g}, [-,-], \partial_\pi)$: DGLA

$\pi_\nu = \pi + \sigma$, s.th. $\sigma \in \mathrm{MC}(\mathfrak{g}) = \{s \in \mathfrak{g}^1 : \partial_\pi s + \frac{1}{2}[s,s] = 0\}$

$\mathrm{Def}(\mathfrak{g}) = \mathrm{MC}(\mathfrak{g})/G(\mathfrak{g})$

$\mathrm{Def} : \mathtt{L}_\infty \to \mathtt{Set}$

THEOREM

If $F : L \to L'$ is a Qiso, then $\mathrm{Def}(F) : \mathrm{Def}(L) \to \mathrm{Def}(L')$ is bijective.

$L = (T_{\mathrm{poly}}(M) = \Gamma(\wedge TM), [-,-]^{\mathrm{SN}}, 0)$: DGLA

$L' = (D_{\mathrm{poly}}(M) \subset \mathcal{L}_\bullet(C^\infty(M)), [-,-]^{\mathrm{G}}, \partial_\mu)$: DGLA

$$\mathrm{L}_\infty(V)$$

$$||$$

$$\{(\pi_1, \pi_2, \pi_3, \ldots), \text{multilin., AS}, |\pi_p| = 2 - p\}$$

$$+$$

seq. conditions

1) $\pi_1^2 = 0$ 2) π_1 der. of π_2 3) π_2 Jacobi id. modulo homotopy

Example: $(\pi_1, \pi_2, 0, \ldots)$ DGLA

$$\pi_p : (sV)^{\vee p} \to sV, \quad \pi : S(sV) \to sV$$

$$\mathrm{L}_\infty(V)$$

$$||$$

$$\{(\pi_1, \pi_2, \pi_3, \ldots), \text{multilin., AS, } |\pi_p| = 2 - p\}$$

$$+$$

seq. conditions

1) $\pi_1^2 = 0$ 2) π_1 der. of π_2 3) π_2 Jacobi id. modulo homotopy

Example: $(\pi_1, \pi_2, 0, \ldots)$ DGLA

$$\pi_p : (sV)^{\vee p} \to sV, \quad \pi : S(sV) \to sV$$

$$\begin{array}{ccc} \mathrm{L}_\infty(V) & \simeq & \mathrm{CoDiff}^1(S(sV)) \\ \parallel & & \parallel \\ \{(\pi_1, \pi_2, \pi_3, \ldots), \text{multilin., AS}, |\pi_p| = 2 - p\} & \simeq & \mathrm{CoDer}^1(S(sV)) \\ + & & + \\ \text{seq. conditions} & \simeq & Q^2 = 0 = [Q, Q] \end{array}$$

1) $\pi_1^2 = 0$ 2) π_1 der. of π_2 3) π_2 Jacobi id. modulo homotopy

Example: $(\pi_1, \pi_2, 0, \ldots)$ DGLA

$$\text{Lod}_\infty(V) \overset{?}{\simeq} \text{CoDiff}^1(T(sV))$$

$$\{(\pi_1, \pi_2, \pi_3, \ldots), \text{ multilin., } \cancel{\text{AS}}, |\pi_p| = 2 - p\} \overset{?}{\simeq} \text{CoDer}^1(T(sV))$$

$$+ \qquad\qquad +$$

$$\text{? seq. conditions} \overset{?}{\simeq} Q^2 = 0 = [Q, Q]$$

1) $\pi_1^2 = 0$ 2) π_1 der. of π_2 3) π_2 Jacobi id. modulo homotopy

Example: $(\pi_1, \pi_2, 0, \ldots)$ DGLodA

THEOREM

$$\Delta(v_1 \otimes \ldots \otimes v_{p-1} \otimes v_p) =$$

$$\sum_{i=1}^{p-2} \sum_{\sigma\in \mathrm{sh}(i,p-1-i)} \pm v_{\sigma(1)} \otimes \ldots \otimes v_{\sigma(i)} \bigotimes v_{\sigma(i+1)} \otimes \ldots \otimes v_{\sigma(p-1)} \otimes v_p$$

$(T(V),\Delta)$: *twisted coassociative coalgebra, i.e.*

$$(\mathrm{id} \bigotimes \Delta)\Delta = (\Delta \bigotimes \mathrm{id})\Delta + (T \bigotimes \mathrm{id})(\Delta \bigotimes \mathrm{id})\Delta$$

THEOREM

$$C_n =$$

$$\sum_{i+j=n+1}\sum_{k\geq j}\sum_{\sigma\in\mathrm{sh}(k-j,j-1)}\chi(\sigma)(-1)^{(k+i-j)(j-1)}(-1)^{j(v_{\sigma(1)}+\ldots+v_{\sigma(k-j)})}$$

$$\pi_i\big(v_{\sigma(1)},\ldots,v_{\sigma(k-j)},\pi_j\big(v_{\sigma(k+1-j)},\ldots,v_{\sigma(k-1)},v_k\big),v_{k+1},\ldots,v_n\big)=0,$$

$\chi(\sigma)$: *Koszul sign*, $\quad n\geq 1$.

Cochain space $= \{$seq. multilin.$\}$

$$[-,-]^{\overline{\text{Stem}}}\ ?$$

$$\mathrm{Lod}_\infty(V)$$

$$\parallel$$

$$\{\text{seq. } \pi = (\pi_1, \pi_2, \ldots) \text{ multilin., } |\pi_p| = 2 - p\}$$

$$+$$

$$[\pi,\pi]^{\overline{\text{Stem}}} = 0?$$

$\rightsquigarrow$ Lod_∞ cohomology ?

Cochain space $= \{$seq. multilin.$\}$ $\simeq$ $\mathrm{CoDer}(T(sV))$

$[-,-]^{\overline{\mathrm{Stem}}}$ $\simeq$ $[-,-]$

$$\begin{array}{ccc} \mathrm{Lod}_\infty(V) & \simeq & \mathrm{CoDiff}^1(T(sV)) \\ \shortparallel & & \shortparallel \\ \{\text{seq. } \pi = (\pi_1, \pi_2, \ldots) \text{ multilin., } |\pi_p| = 2 - p\} & \simeq & \mathrm{CoDer}^1(T(sV)) \\ + & & + \\ [\pi, \pi]^{\overline{\mathrm{Stem}}} = 0 & \simeq & Q^2 = 0 = [Q, Q] \end{array}$$

$\rightsquigarrow$ Lod_∞ cohomology

Stem bracket

Theorem

$$[\pi,\rho]^{\overline{\text{Stem}}} = \sum_{q\geq 1}\sum_{s+t=q+1}(-1)^{1+(s-1)\langle e_1,\rho\rangle}[\pi_s,\rho_t]^{\text{Stem}},$$

$$[A,B]^{\text{Stem}} = j_A B - (-1)^{\langle (A,a),(B,b)\rangle} j_B A,$$

$$j_B A = (-1)^{\langle A,B\rangle}\sum_{\substack{I\cup J\cup K=N^{(a+b+1)}\\ I,J<K,\ |J|=b}}(-1)^{\langle B,V_I\rangle+b|I|}(-1)^{(I;J)}\varepsilon_V(I;J)$$

$$A(V_I\otimes B(V_J\otimes v_{k_1})\otimes V_{K\setminus k_1})$$

THEOREM

$\overline{[\pi, -]^{\text{Stem}}}$ *encodes*

- *graded Loday (DT), graded Lie (LMS), graded Poisson, graded Jacobi (GM, up to isom) cohomologies*
- *Loday infinity, Lie infinity (FP), 2n-ary graded Loday and Lie (MV, VV) cohomologies*

CATEGORY THEORETICAL APPROACH

- Sets $\rightsquigarrow$ categories
- Maps $\rightsquigarrow$ functors
- Equations $\rightsquigarrow$ natural isomorphisms + coherence laws

1. vs $\simeq$ linear set $\rightsquigarrow$ 2vs $\simeq$ linear category
2. LA $\simeq$ vs + bilinear map + AS, Jacobi

   $\rightsquigarrow$

   L2A $\simeq$ 2vs L + bilinear functor $[-,-]$ + trilinear natural isomorphism $J : [-,[-,-]] + \circlearrowleft \Rightarrow 0$ + coherence laws

   Semistrict L2A [Baez, Crans, '04], weak L2A [Roytenberg, '07]

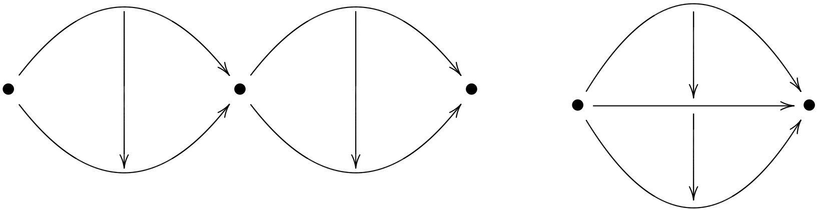
2 category (rough description):

- **objects** A, **1-morphisms** $f : A \to B$, **2-morphisms** $\alpha : f \Rightarrow f'$ (homological algebra, string theory)
- **vertical composition**: $f, f', f'' \in \mathrm{Hom}(A, B)$, $\alpha : f \Rightarrow f'$, $\beta : f' \Rightarrow f''$,

$$\alpha \bullet \beta : f \Rightarrow f''$$

- **functorial horizontal composition**: $\circ : \mathrm{Hom}(A, B) \times \mathrm{Hom}(B, C) \to \mathrm{Hom}(A, C)$ is a bifunctor, i.e. is s.th. if $\alpha : f \Rightarrow f'$, $\beta : g \Rightarrow g'$, then

$$\alpha \circ \beta : f \circ g \Rightarrow f' \circ g'$$



- Objects: L2A $(L, [-,-], J)$, $(L', [-,-]', J')$, $\ldots$
- 1-morphisms:
  lin fun $F : L \to L'$
  bilin natural iso $F_2 : [-,-]' \circ (F \otimes F) \Rightarrow F \circ [-,-]$
  respect of Jacobiators
- 2-morphisms:
  lin natural transfo $\theta : F : L \to L' \Rightarrow G : L \to L'$
  respect of F_2 and G_2

2vs: $s, t : L_1 \to L_0 \, + \, 1, \circ$;   2term: $\pi_1 : V_1 \to V_0$; $\pi_1 = t|_{\ker s}$

Lie2Alg

- Objects: L2A $(L,[-,-],J)$, $(L',[-,-]',J'),\dots$
- 1-morphisms:

  lin fun $F:L\to L'$

  bilin natural iso $F_2:[-,-]'\circ(F\otimes F)\Rightarrow F\circ[-,-]$

  respect of Jacobiators
- 2-morphisms:

  lin natural transfo $\theta : F:L\to L'\Rightarrow G:L\to L'$

  respect of F_2 and G_2

2vs: $s,t:L_1\to L_0\ +\ 1,\circ$;   2term: $\pi_1:V_1\to V_0$; $\pi_1=t|_{\ker s}$

$\mathtt{Lie2Alg}\simeq\mathtt{2TermL}_\infty$

THEOREM

$$\mathtt{Lod2Alg} \simeq \mathtt{2TermLod}_\infty$$

Suppression of AS = destruction of simplifications

Conceptual approach to and explicit formulae for 1- and 2-morphisms

Conjecture 1: $\mathrm{LnA} \simeq \mathrm{nTermL}_\infty$

L2A	$\sim$	cat L	fun $[-,-]$	trans J	coh	
L3A	$:\sim$	2-cat L	2-fun $[-,-]$	2-trans J	2-mod I	?coh?
$\updownarrow$		$\updownarrow$ (1)	$\updownarrow$ (2)	$\updownarrow$ (3)	$\updownarrow$ (4)	$\updownarrow$ (5)
$3\mathrm{TermL}_\infty$	$\sim$	V_i, π_1, C_1	π_2, C_2	π_3, C_3	π_4, C_4	C_5

Answer 1a: (5) involves more than 100 terms

Answer 1b: (2) is " $[-,-]$ respects $\circ \leftrightarrow C_2$" and holds true only under two (acceptable) conditions

Fact: $\texttt{Vect}$ n-$\texttt{Cat}$ is a symmetric monoidal category

$$\begin{array}{ccc} L \times L' & \to & L'' \\ \boxtimes \downarrow & \nearrow & \\ L \boxtimes L' & & \end{array}$$

! $\boxtimes : L \times L' \to L \boxtimes L'$ is not an n-functor ! $(\star)$

Conjecture 2: Use

$\mathcal{N} : \texttt{VectCat} \to s(\texttt{Vect})$, $N : s(\texttt{Vect}) \to C^+(\texttt{Vect})$,

$\mathrm{EZ} : N(S) \otimes N(T) \leftrightarrow N(S \otimes T) : \mathrm{AW}$

Answer 2: Obstruction = defect $(\star)$

Operadic approach

5 definitions:

classical, partial compositions, type of algebras, functorial, combinatorial

algebra $\rightarrow$ matrices, operad $\rightarrow$ algebra

operad:

- vs $P(n), n \in \mathbb{N}^*$, of abstract n-ary operations - T_n

5 definitions:

classical, partial compositions, type of algebras, functorial, combinatorial

algebra $\to$ matrices, operad $\to$ algebra

operad:

- vs $P(n), n \in \mathbb{N}^*$, of abstract n-ary operations - T_n
- S_n-module structure on $P(n)$
- linear composition maps

$$\gamma_{i_1 \ldots i_k} : P(k) \otimes P(i_1) \otimes \ldots \otimes P(i_k) \to P(i_1 + \ldots + i_k),$$

which are associative, respect the S-action (and possibly the unit)

Example:

AA

$P(1) = \mathbb{K}\ T_1$

$P(2) = \mathbb{K}\ T_2(12) + \mathbb{K}\ T_2(21)$ (encoding of symmetries)

$P(3) = \mathbb{K}\ T_2(T_2, T_1) + \mathbb{K}\ T_2(T_1, T_2)$ (encoding of defining relations)

$P(n) = \mathbb{K}[S_n]$

General case:

type of algebras

- generating operations with symmetries $\rightsquigarrow$ S_n-module $M(n)$
- defining relations $\rightsquigarrow$ ideal (R)

operad $P = \mathcal{F}(M)/(R)$

A: D(GA)A, C: D(GA)C, suspensions + reductions understood

- $\mathrm{Hom}_{\mathrm{alg}}(T(C), A) \simeq \mathrm{Hom}_{\mathbb{K}}(C, A) \simeq \mathrm{Hom}_{\mathrm{coalg}}(C, T^c(A))$

  ΩC: DA cobar complex, BA: DC bar complex

  $d_2(a_1 \otimes \ldots \otimes a_n) = \sum \pm a_1 \otimes \ldots \otimes \mu(a_i, a_{i+1}) \otimes \ldots \otimes a_n$

- $\mathrm{Hom}_{\mathrm{DA}}(\Omega C, A) \simeq \mathrm{TW}(C, A) \simeq \mathrm{Hom}_{\mathrm{DC}}(C, \mathrm{BA})$
- $\mathrm{Qiso}(\Omega C, A) \simeq \mathrm{Kos}(C, A) \simeq \mathrm{Qiso}(C, \mathrm{BA})$

  $d_\alpha : C \otimes A \stackrel{\Delta \otimes \mathrm{id}}{\to} C \otimes C \otimes A \stackrel{\mathrm{id} \otimes \alpha \otimes \mathrm{id}}{\to} C \otimes A \otimes A \stackrel{\mathrm{id} \otimes \mu}{\to} C \otimes A$

$\Omega\, BA \stackrel{\sim}{\to} A$: bar-cobar resolution of A

Quadratic data: V, $R \subset V^{\otimes 2}$ (e.g. $R = \langle v \otimes v' - v' \otimes v \rangle$)

Quadratic algebra: $A(V,R) = T(V)/(R) \supset \mathbb{K} \oplus V$ (e.g. $S(V)$)

Quadratic coalgebra:

$$C(V,R) = \mathbb{K} \oplus V \oplus \bigoplus_{k\geq 2} \bigcap_{i+j+2=k} V^{\otimes i} \otimes R \otimes V^{\otimes j} \subset T^c(V)$$

Koszul dual coalgebra of $A(V,R)$:

$$A^{¡} = C(sV, s^2 R) \supset \mathbb{K} \oplus sV$$

$\mathrm{TW}(A^{¡}, A) \ni \alpha : A^{¡} \twoheadrightarrow sV \to V \rightarrowtail A$

If $\alpha \in \mathrm{Kos}(A^{¡}, A)$, then

$$\Omega A^{¡} \xrightarrow{\sim} A$$

Extension of this minimal model of Koszul algebras [Priddy, '70] to Koszul operads [Ginzburg, Kapranov, '94]

$\mathrm{End}(V)$: endomorphism operad on a vs V

$P \to \mathrm{End}(V)$: operadic morphism, **type P algebra structure on V**

If P is a Koszul operad,

$$\begin{array}{ccc} \Omega P^{\text{i}} & \stackrel{\sim}{\to} & P \\ & \searrow & \downarrow \\ & & \mathrm{End}(V) \end{array}$$

$\mathrm{End}(V)$: endomorphism operad on a vs V

$P \to \mathrm{End}(V)$: operadic morphism, **type P algebra structure on V**

If P is a Koszul operad,

$$\begin{array}{ccc} P_\infty := \Omega P^{¡} & \stackrel{\sim}{\to} & P \\ & \searrow & \downarrow \\ & & \mathrm{End}(V) \end{array}$$

From $\Omega\mathcal{L}ie^{¡}$ and $\Omega\mathcal{L}od^{¡}$ we recover L_∞ and Lod_∞ structures on V

- Leibniz (Loday) algebras
  - ⋆ Origin: periodicity phenomena in K-theory
  - ⋆ Examples:
    1. Courant-Dorfman bracket, Courant algebroid bracket
    2. Loday bracket associated with a Nambu-Poisson structure
    3. Derived brackets
- Loday infinity algebras
  - ⋆ Origin: universal GLA and cohomologies
  - ⋆ Examples:
    1. Fulton-MacPherson type compactification [Kontsevich, ’03] [Merkulov, ’09]
    2. Formal deformation of a DGLodA and derived brackets [Uchino, ’09]

SUPERGEOMETRIC APPROACH

Ginzburg-Kapranov, '94:

P_∞-algebra on $V \rightleftharpoons Q \in \mathrm{Der}_1(\mathcal{F}^{\mathrm{gr}}_{P^!}(sV^*)), Q^2 = 0$

Example:

L_∞-algebra $\rightleftharpoons$ homological vf on a pointed formal smfd

Geometric 'example':

L_∞-algebroid $\rightleftharpoons$ homological vf on a split $\mathbb{N}$-mfd

Particular case:

LAD$\rightleftharpoons$ homological vf on a split smfd

Requirements

Find a concept of Loday algebroid that

- is close to the notion of Lie algebroid
- contains Courant algebroids as special case
- reduces to a Loday algebra over a point
- includes a differentiability condition on both arguments,

and interpret it as homological vf on a supercommutative mfd

'Definition': A Loday algebroid is a Loday bracket $[-,-]$ on sections of a vb E together with a left and right anchor

J. Grabowski, G. Marmo:

- If $\mathrm{rk}(E) = 1$, $[-,-]$ is AS and 1st order
- If $\mathrm{rk}(E) > 1$, $[-,-]$ is 'locally' a LAD bracket

'No' new examples $\rightsquigarrow$ modify 'definition'

$\rho : \Gamma(E) \to \mathrm{Der}\, C^\infty(M)$, $C^\infty(M)$-linear

$\partial_x(fY) = \partial_x f\ Y + f\ \partial_x Y$, $\partial_x^2(fY) = \partial_x^2 f\ Y + 2\partial_x f \partial_x Y + f\ \partial_x^2 Y$

$[X, fY] = f[X, Y] + \rho(X)f\ Y$

$[X^i e_i, fY^j e_j] =$
$X^i a_{ij}^k f Y^j\ e_k + X^i \rho_i^a \partial_a f\ Y^j e_j + X^i \rho_i^a\ f\ \partial_a Y^j\ e_j - Y^i \rho_i^a \partial_a X^j\ e_j$

$\rho(X)(\mathrm{d} f \otimes Y) = X^i \rho_{ij}^{ak} \partial_a f\ Y^j e_k$

$\rho : \Gamma(E) \stackrel{C^\infty(M)-\mathrm{lin}}{\longrightarrow} \Gamma(TM) \otimes_{C^\infty(M)} \mathrm{End}_{C^\infty(M)} \Gamma(E)$

$\rho : E \to TM \otimes \mathrm{End}\, E$

Cohomology theory $\rightsquigarrow$ representation, traditional left anchor

DEFINITION

A Loday algebroid (LodAD) is a Loday bracket on sections of a vb $E \to M$ together with two bundle maps $\rho : E \to TM$ and $\alpha : E \to TM \otimes \mathrm{End}\, E$ such that

$$[X, fY] = f[X, Y] + \rho(X)f\ Y$$

and

$$[fX, Y] = f[X, Y] - \rho(Y)f\ X + \alpha(Y)(\mathrm{d} f \otimes X).$$

EXAMPLES

- Loday algebra
- (twisted) Courant-Dorfman ($TM \oplus T^*M$)
- Grassmann-Dorfman ($TM \oplus \wedge T^*M$ or $E \oplus \wedge E^*$)
- Leibniz algebroid associated to a Nambu-Poisson structure
- Courant algebroid
- ...

$(E, [-,-], \rho) \rightleftharpoons Q \in \mathrm{Der}_1(\Gamma(\wedge E^*), \wedge), Q^2 = 0$

$(E, [-,-], \rho) \to$ LAD cohomology operator ∂_ρ

∂_ρ is the C-E operator restricted to
$\wedge_{C^\infty(M)}(\Gamma(E), C^\infty(M)) = \Gamma(\wedge E^*)$

∂_ρ is the Loday operator restricted to

$$\mathrm{Lin}_{C^\infty(M)} \quad (\Gamma(E), C^\infty(M)) = \Gamma(\otimes E^*)$$

$(E, [-,-], \rho) \rightleftharpoons Q \in \mathrm{Der}_1(\Gamma(\wedge E^*), \wedge), Q^2 = 0$

$(E, [-,-], \rho) \to$ LAD cohomology operator ∂_ρ

∂_ρ is the C-E operator restricted to
$\wedge_{C^\infty(M)}(\Gamma(E), C^\infty(M)) = \Gamma(\wedge E^*)$

∂_ρ is the Loday operator restricted to

$$D(\Gamma(E), C^\infty(M)) \qquad =: D(E)$$

$(E, [-,-], \rho) \rightleftharpoons Q \in \mathrm{Der}_1(\Gamma(\wedge E^*), \wedge),\ Q^2 = 0$

$(E, [-,-], \rho) \to$ LAD cohomology operator ∂_ρ

∂_ρ is the C-E operator restricted to $\wedge_{C^\infty(M)}(\Gamma(E), C^\infty(M)) = \Gamma(\wedge E^*)$

∂_ρ is the Loday operator restricted to

$$D(\Gamma(E), C^\infty(M)) \qquad =: D(E)$$

$$(D \pitchfork \Delta)(X_1, \ldots, X_{p+q})$$

$$= \sum_{\sigma \in \mathrm{sh}(p,q)} \mathrm{sign}(\sigma) D(X_{\sigma_1}, \ldots, X_{\sigma_p})\, \Delta(X_{\sigma_{p+1}}, \ldots, X_{\sigma_{p+q}})$$

$(E, [-,-], \rho, \alpha) \rightleftharpoons Q \in \mathrm{Der}_1(D(E), \pitchfork), Q^2 = 0$

$\mathcal{D}^k(E) \subset D^k(E)$: k-DO of deg 0 in last section and tot deg $k-1$

$\mathcal{D}^k(E) \pitchfork \mathcal{D}^\ell(E) \subset \mathcal{D}^{k+\ell}(E)$: reduced shuffle algebra

$\partial_\rho \mathcal{D}^k(E) \subset \mathcal{D}^{k+1}(E)$: LodAD subcomplex

$(E, [-,-], \rho, \alpha) \rightleftharpoons Q \in \mathrm{Der}_1(\mathcal{D}(E), \pitchfork), Q^2 = 0$

$$\rho(X)f := \langle Qf, X\rangle$$

$$\langle \ell, [X,Y]\rangle = \langle X, Q\langle \ell, Y\rangle\rangle - \langle Y, Q\langle \ell, X\rangle\rangle - (Q\ell)(X,Y)$$

$Q(\mathcal{D}^2(E)) \not\leftrightarrow Q(\mathcal{D}^0(E) \oplus \mathcal{D}^1(E))$

$\mathcal{D}er_1(\mathcal{D}(E), \pitchfork) \subset \mathrm{Der}_1(\mathcal{D}(E), \pitchfork)$: deg 1 der + appropriate rel

$(E, [-,-], \rho, \alpha) \rightleftharpoons Q \in \mathcal{D}er_1(\mathcal{D}(E), \pitchfork), Q^2 = 0$

$(E, [-,-], \rho, \alpha) \rightleftharpoons [Q] \in \mathcal{D}er_1(\mathcal{D}(E), \pitchfork), Q^2 = 0$

THEOREM

There is a 1-to-1 correspondence between LodAD structures on a vb E and equivalence classes of homological vfs

$$Q \in \mathcal{D}er_1(\mathcal{D}(E), \pitchfork), Q^2 = 0$$

of the supercommutative mfd $(\mathcal{D}(E), \pitchfork)$.

Remarks: Homological vf $Q \rightsquigarrow$

- Cartan calculus for $(D(E), \pitchfork)$
- LodAD bracket is derived bracket given by gLa $(\mathrm{Der}(D(E), \pitchfork), [-,-]_c)$ and its interior der $[Q, -]_c$

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Thank you!